

Teaching Through Investigations to Empower Students and Build Identity and Agency

K-5

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How might teaching through investigations deepen students' math understanding as well as their curiosity, confidence, identity, and math joy?

Highlighted Investigations

Adding fractions with like denominators

Context: *Enemy Pie* by Derek Munson

Investigate

Dad cut the enemy pie in sixths. Show it with your pattern blocks.



Dad ate 2 sixths of the pie and Jeremy ate one sixth of the pie. How many sixths did they eat?

2 sixths + 1 sixth = _____

Mathematicians don't write out the words like we did. Turn and share how a mathematician would write this.

$$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$$

Pose more problems.

$$\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$$

$$\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

$$\frac{2}{6} + \frac{2}{6} = \frac{4}{6}$$

What do you notice?

Why is it happening?

Why are the denominators staying the same?

Why are the numerators changing?

If Dad had $\frac{3}{6}$ and Jeremy had $\frac{1}{6}$, how many sixths do you think they would have eaten?

Try it to see if you were right.

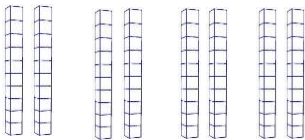
How can you add fractions without using the models?

Multiplying a single-digit number by a multiple of 10

Context: *Dragons Love Tacos* by Adam Rubin

The dragons had 4 trays of 20 tacos. How many tacos did they have?

- Show me 20 tacos using your base -ten rods.
- What would 4×20 be?
- Show it and record the equation.



$$4 \times 20 = 80$$

Trays	Tacos	Total
4	$20 = 80$	
3	$20 = 60$	
3	$30 = 90$	
4	$30 = 120$	

- What do you notice?
- Why is this happening?
- Predict $7 \times 20 = \underline{\quad}$.
- Model it. Were you correct?
- What is the rule? Why does it work?



Subtracting a multiple of ten from a multiple of ten

Context: *Pete the Cat Snow Daze* by James Dean

The Snowball Fight

- If Pete made 40 snowballs and threw 10 of them. How many would he have left?
- How can you show what happened in the problem?

$$40 - 10 = 30$$

tens	ones

The Snowball Fight

Pose more problems for students to model and solve.

Record the equations.

$$40 - 10 = 30$$

$$60 - 20 = 40$$

$$80 - 30 = 50$$

$$50 - 30 = 20$$

What do you notice?

Does it make sense? Why or why not?

Predict $70 - 30$. Try it to see if you were right.

Can you find the solution without using a model? How?

Exploring Properties

Investigating the Commutative Property with Models

2 plates of 5 brownies 5 plates of 2 brownies



$$2 \times 5 = 10$$



$$5 \times 2 = 10$$

Investigating the Commutative Property with Models

- 2 plates of 5 brownies $2 \times 5 = 10$
- 5 plates of 2 brownies $5 \times 2 = 10$
- 2 plates of 3 brownies $2 \times 3 = 6$
- 3 plates of 2 brownies $3 \times 2 = 6$
- 2 plates of 4 brownies $2 \times 4 = 8$
- 4 plates of 2 brownies $4 \times 2 = 8$

What do you notice?

Do you think it will always happen? Why or why not? If $2 \times 6 = 12$, what is the product of 6×2 ? Try it. What is the rule?

Making Sense of the Distributive Property

- There were 6 plates with 7 cookies on each plate.
- How many cookies were there?
- Draw a model.
- How would you find the total?
- Could you multiply in two groups?



$$(2 \times 7) + (4 \times 7) = 6 \times 7$$

Understanding Our Counting Numbers

Observing Numbers

- What is happening?
- Does that make sense?
- Predict what a tower of 7 cubes looks like.
- Try it to see if you were right.
- What do you notice about numbers as you count?



MATH PRACTICE

Making Sense of 2-digit Place Value

Making Sense of Place Value

1. Have partners count out 25 unifix cubes.
2. Have them put the cubes on ten frames and then check the total.



How did you find the total each time? Do the different ways of counting give you the same answer? Why? How many ten frames did you fill? How many ones do you have left over? Create a chart to record students' data.

Observe and Consider

Number of Counters	Filled Ten Frames	Leftover Ones
25	2	5
32	3	2
14	1	4
26	2	6
38	3	8

What do you notice? What do you wonder? Predict how many tens and ones are in 38. Try it and see if you were right. Can you figure out how many ten frames will be filled without actually filling them? How?

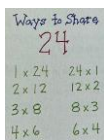
MATH PRACTICE

Understanding Prime and Composite Numbers

Context: *Bean Thirteen* by Matthew McElligott

Sharing Beans

- Could they have fairly shared 24 beans?
- Could you use your tiles to make arrays to show how Ralph and Flora might have shared the beans?
- Do you think there is more than one way?



MATH BOOK

Exploring Prime and Composite

What do you notice? How are the posters alike? How are they different? Without giving an explanation, rearrange the posters into 2 groups: ones with only one area model and ones with more than one area model. Label the groups as prime or composite. Have student teams come up with the definitions of prime and composite.



MATH BOOK

Exploring Odd and Even Numbers

Context: *Even Steven and Odd Todd* by Kathryn Cristaldi

Investigate: Odd or Even?

Explore the numbers for Todd by making chains of linking cubes and trying to break them into 2 chains of equal parts.

What do you notice?

Give each pair a 1-20 chart and have them highlight all of the numbers that can be broken into 2 equal parts.

Even Steven	Odd Todd
10	3
6	9
12	5
4	7
2	
8	



Record and discuss results.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

Which numbers can be split into 2 equal chains? Are you noticing a pattern? Tell your partner what you see.

Not all of the numbers on the 1-20 chart were in the story. What other numbers might belong with Even Steven? How will you test to see if they belong? The numbers you highlighted are called even numbers. What is an even number?

Understanding the Concept of Area

Context: *The Raft* by Jim LeMarche

Investigate: Exploring Area

- Students use grid paper to outline rafts of different sizes.
- Students explore the sizes of the rafts, record the data, and look for patterns to find the rule.

Length in Inches	Width in Inches	Area of the Rectangle
5	3	15 squares
3	4	12 squares
4	2	8 squares

What do you notice?

Try a few more. Does this keep happening?

Understanding the Concept of Volume

Context: *Crayon Man: The True Story of the Invention of the Crayola Crayon* by Natascha Biebow

Investigate: A Box of Crayons

Once a product is created, it has to be packaged for sale. How did they package crayons?

Is it important that the box is just the right size? Why or why not?

How did they know how big to make the box?

How big is a crayon?

Box templates are filled with centimeter cubes to find the volume.



What if we put 16 crayons in the box in 2 layers? Or 24 in 3 layers? What is the volume?

Investigate: Boxes of Crayons

Length of box (cm)	Width of box (cm)	Height of box (cm)	Total number of cm cubes
9	8	1	72
9	8	2	144
9	8	3	216

What do you notice?

What patterns do you see?

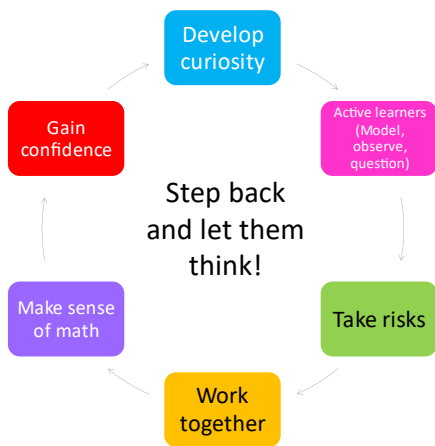
Do you have to fill a box and count the squares?

How else might you find the volume?



Thinking About the Investigations

- Who did the thinking?
- How did the materials help?
- How did my recordings help?
- What questions did I ask to prompt discovery?
- What are the benefits of stepping back and letting students discover math ideas through investigations?



Resources

For more investigations:

www.MathinPractice.com

www.MathbytheBook.com

For additional resources, visit Sue's website at <https://www.qualityteacherdevelopment.com/>

Follow Sue on Twitter @SueOConnellMath

Join the Math in Practice Facebook group!

Coming soon from Hand2Mind: Navigating Numeracy K-5 Center Kits - <https://www.hand2mind.com/supplemental-curriculum/math/navigating-numeracy-learning-progression-centers>

For literature to launch math lessons:

Padlet of K-2 Children's Literature Related to Math Skills/Concepts

<https://padlet.com/sueoc46/j3uevoy154m1fuhb>

Padlet of Grades 3-5 Children's Literature Related to Math Skills/Concepts

<https://padlet.com/sueoc46/who7w8089bm6r6k2>

Padlet of K-5 Culturally Diverse Children's Literature with Math Connections

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Download a list of 120 children's books and associated math topics at:

www.MathbytheBook.com