

From Models to Discovery: Helping Students Make Sense of Mathematics K-5

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"Sometimes, telling kids where they are going spoils the journey." Dylan Wiliam

Step Back and Let Them Think

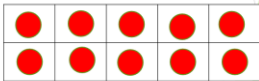
What investigations, models, and questions help K-5 students make sense of mathematics?

Understanding Teen Numbers

Grandma's Tiny House by JaNay Brown-Wood

How many melons did the nieces bring to the feast?

Show 14.



Record and discuss.

Total Counters	On the ten frame	How many more?
14	10	4
15	10	5
12	10	2
17	10	7

What do you notice?
Tell your partner what you think 13 counters will look like on a ten frame.
Try it. Were you right?
Can you know how many more than 10 without using your counters? How?

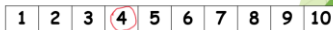
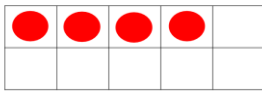
Understanding the Concept of +1 or 1 More

One More Dino on the Floor by Kelly Starling Lyons

Show 3 dinos.

Add 1 more on the floor.

$$3 + 1 = 4$$



Record and discuss results.

$$3 + 1 = 4$$

$$5 + 1 = 6$$

$$7 + 1 = 8$$

$$2 + 1 = 3$$

What do you notice?
What is $4 + 1$? How do you know?
Try it. Were you right?
Could you add 1 without using the counters?
How?
What happens when you add 1 to a number?

Understanding Place Value with Two-digit Numbers

Too Many Mangos by Tammy Paikai

How many bags of 10 mangos?

- Have partners count out 25 unifix cubes.
- Have them put the cubes on ten frames and then check the total.



How many bags of ten mangos?
How many mangos are left and not in a bag?
Create a chart to record students' data.



Observe and Consider

Number of Counters (total mangos)	Filled Ten Frames (bags of 10 mangos)	Leftover Ones (leftover mangos)
25	2	5
32	3	2
14	1	4
26	2	6

What do you notice?
What do you wonder?
Predict how many groups of ten and leftover ones are in 38.
Try it and see if you were right.

Can you figure out how many groups of ten without actually filling the ten frames? How?

Mentally Knowing 10 Less than a 3-digit Number

Too Many Pumpkins by Linda White

A truck with 241 pumpkins hit a bump and 10 pumpkins spilled out onto the ground. How many pumpkins were still in the truck?

Retell it.

Visualize it.

How do you show this? Which block do you remove? Why? Do it and show the number of pumpkins on the truck now.

10 pumpkins fell off the truck.

Hundreds	Tens	Ones
2	4 3	1

Do more trials and record the data.

241	379	219	425
231	369	209	415

*What do you notice?
What digits changed? Why?
Predict: What is 10 fewer than 267?
Check your prediction with a model.
Tell your partner how to find 10 less than a 3-digit number.*

Understanding the Concept of Division and Making Connections to Multiplication

Around Our Way on Neighbors' Day by Tameka Fryer Brown

One neighbor brought a plate of 12 cookies to the block party. How could the children share them fairly?

Create a model to show how you might share them so each child would get the same amount of cookies?

What do you notice?

$12 \div 2 = 6$
 $12 \div 3 = 4$
 $12 \div 4 = 3$
 $12 \div 6 = 2$

Adding Fractions with Unlike Denominators

Picture Pie by Ed Emberley

Art is created with fractional pieces? What is the value of the art?

What is the value of the flower?

► Why might it help to think of all the parts as eighths?

► What might be a rule for adding fractions when the denominators are different?

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = n$

$\frac{4}{8} + \frac{2}{8} + \frac{4}{8} = \frac{10}{8}$ or $1\frac{2}{8}$

Discovering the Formula for Volume

Crayon Man: The True Story of the Invention of the Crayola Crayon by Natascha Biebow

What size box do we need for 8 crayons? 16 crayons? 24 crayons?

Create boxes by folding centimeter grid paper, fill with centimeter cubes, and record the results.

Observing and Analyzing Data

Length of Box in cm	Width of Box in cm	Height in cm	Volume In cm^3
9	8	1	72 cm^3
9	8	2	144 cm^3
9	8	3	216 cm^3

What do you notice?

Predict: How many centimeter cubes it would take to fill a box with 4 layers.

Could you figure the volume without filling the box with cubes? How?

What is a rule for finding the volume?

What do these tasks have in common?

- Visual
- Context
- Deep questioning
- Math talk
- Ask students to make sense of ideas
- Ask students to generalize
- Focus on discovery rather than telling

Questions to Ask

- What do you notice?
- Why is it happening?
- Does it make sense?
- Can you predict...?
- What is the rule?

How do we align our instruction to our standards?

- Step back and let them think.
- Give math a context.
- Make math visual.
- Get them talking.
- Watch and listen to students and adjust our teaching to meet their needs.

From *Math in Practice: A Guide for Teachers* by Susan O'Connell (www.MathinPractice.com)

For more grade-specific investigations, see the *Math in Practice* series (www.MathinPractice.com).

For more investigations linked to a context from children's literature, see the *Math by the Book* series (www.MathbytheBook.com).

Teacher Resource Books by Sue O'Connell and Colleagues

Published by Heinemann (www.heinemann.com)

Math by the Book (www.mathbythebook.com)

This K-5 series explores the teaching of math concepts through children's literature. Lessons, word problems, discussion questions, differentiation ideas, and practice tasks are all included to teach grade-specific skills and concepts through the story context. There is a book for each grade level K-5, including a wealth of online resources, and each book includes ideas for twenty skills taught during that year.

Math in Practice (www.mathinpractice.com)

This K-5 series is filled with lesson ideas, instructional strategies, practice tasks, and many online printable resources to make teaching K-5 math more meaningful and more fun. There is a book for each grade level K-5 that contains grade-specific activities that span the math standards for the whole year, as well as a *Guide for Teachers* filled with instructional strategies and an *Administrator's Guide* for math coaches and administrators.

Putting the Practices into Action - Implementing the Common Core Standards for Mathematical Practice K-8

with John SanGiovanni

The Standards for Math Practice are the heart and soul of our teaching practice. This book explains each standard in teacher-friendly terms and highlights practical activities to make the standards come alive in classrooms. It contains PLC study group questions and online resources.

Mastering the Basic Math Facts for Addition and Subtraction

Mastering the Basic Math Facts for Multiplication and Division

with John SanGiovanni

Through investigations, discussions, visual models, children's literature, and hands-on explorations, students explore the math operations, and through engaging, interactive practice achieve fluency with basic facts. A teacher-friendly CD filled with customizable activities, templates, recording sheets, and teacher tools simplifies your planning and preparation. Over 450 pages of reproducible forms are included in English and Spanish translation.

The Math Process Standards Series

Each book in this series is a practical guide for helping students refine their skills in the highlighted math process (problem solving, communication, reasoning, representations, connections). You will find specific teaching strategies and tips to help all students strengthen their skills.

Introduction to Problem Solving

Introduction to Communication

Introduction to Representation

Introduction to Reasoning and Proof

Introduction to Connections

All books in this series are available for Grades PK-2, Grades 3-5, and Grades 6-8.

For additional resources, visit Sue's website at www.qualityteacherdevelopment.com

Follow Sue on Twitter @SueOConnellMath

Gather strategies and ideas on Facebook by liking Sue's Facebook page (Quality Teacher Development) or joining the *Math in Practice* Facebook group!