

# Moving Beyond Memorization: Making Sense of Fractions Through Discovery

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## What does proficiency with fractions look like?

Consider the following problems:

Blake is making brownies and needs  $\frac{3}{4}$  cup of milk, but there is no measuring cup that says  $\frac{3}{4}$ . How can she be sure she has just the right amount of milk?



Molly, Liam, and Bailey ordered a pepperoni pizza and they ate the whole thing!

Liam ate more than the others.

What fraction of the pizza could each child have eaten? Justify your answers.

Katie sold 30 boxes of Girl Scout Cookies.

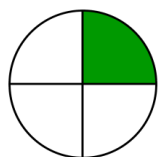
$\frac{2}{5}$  of them were Thin Mints.

How many boxes of Thin Mint Cookies did she sell? Explain how you know.

**Our goal is for our students to develop a comprehensive, multi-faceted knowledge of fractions, not simply to memorize a series of procedures.**

### Introducing Fraction Notation

What part of this circle is shaded green?  
"a fourth"

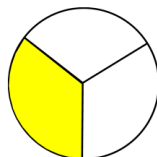


$$\frac{1}{4}$$

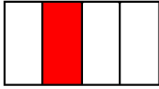
Turn and talk:  
What do you think each number represents? Explain your thinking.

### Ask, Don't Tell

What fraction would you write to show the part of this circle that is shaded yellow?  
Explain the numbers in your fraction.



### Try More Examples



Write the fraction that shows the part of this rectangle that is colored red.  
What does each number in your fraction represent?

### Let Them Try It

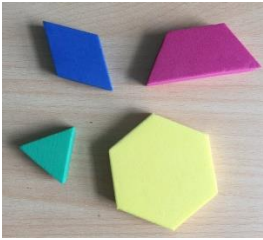


Write the fraction that shows the part of this circle that is colored green.  
What does each number in your fraction represent?

## Extending Understanding

Moving Beyond Circles and Rectangles

Provide students with pattern blocks.



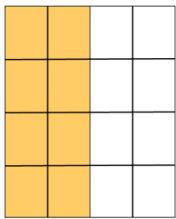
*Can you talk about any of the blocks using fractions?*

Allow students to explore and share their findings.

Share and discuss their findings.

## Discovering Equivalence Through Paper Folding

### Exploring Equivalence



How would you describe one side of your paper?

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{1}{2} = \frac{4}{8}$$

$$\frac{1}{2} = \frac{8}{16}$$

MATH PRACTICE

Has the size of  $\frac{1}{2}$  of your paper changed?

What has changed about the half of your paper?

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{1}{2} = \frac{4}{8}$$

$$\frac{1}{2} = \frac{8}{16}$$

Can you use your data to develop a definition of equivalent fractions?

Can you use your data to develop a rule for generating equivalent fractions?

## Further Investigations of Equivalence

### Equivalent Fractions



- Draw a rectangle.
- Split it into fourths with vertical lines.
- Shade  $\frac{1}{4}$ .
- Divide the rectangle again with a horizontal line.
- *What fractional part is shaded?*
- *Write an equality to show it.*

$$\frac{1}{4} = \frac{2}{8}$$

### Equivalent Fractions



Record on the board.

$$\frac{1}{4} = \frac{2}{8}$$

$$\frac{1}{4} = \frac{3}{12}$$

$$\frac{1}{4} = \frac{4}{16}$$

$$\frac{1}{4} = \frac{5}{20}$$

*What do you notice? What do you wonder?  
Will it always happen? Explain your thinking.*

Try it with a different unit fraction (e.g.,  $\frac{1}{3}$ ,  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{8}$ ).

Model your fraction and find some equivalent fractions.

Be ready to present your findings.

*What do you notice?*

*Is there a way you could find the equivalent fractions without creating the models? Explain.*

*What is the rule?*

## Investigating with Non-unit Fractions

Give each team a non-unit fraction to model and then find equivalent fractions (e.g.,  $\frac{2}{3}$ ,  $\frac{3}{5}$ ,  $\frac{5}{6}$ ,  $\frac{3}{8}$ ).

Have teams present their findings.

*What do you notice?*

*Does it make sense?*

*How can you tell if fractions are equivalent? What is always true for equivalent fractions?*

*What is the rule for generating equivalent fractions?*

## Talk About It/Write About It

What does it mean for fractions to be equivalent?

Agree or disagree?

$$\frac{1}{5} = \frac{5}{20}$$

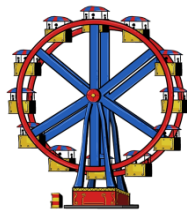
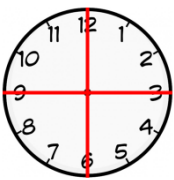
Justify your answer.

## Changing Fraction Models Based on Context

Katie walked  $\frac{1}{2}$  mile to school.

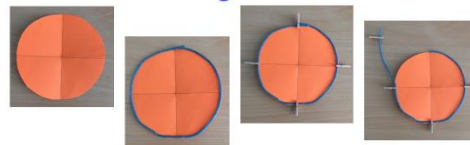
How might you show that?

### Connecting Area Models to Measurement Models

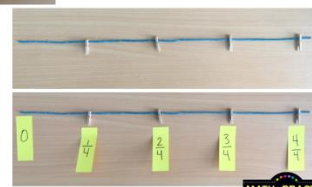


When the minute hand starts at 12 and travels around the clock face, when does it reach one quarter past the hour? Why?

### Connecting Fraction Models



What do you notice?  
How is the fraction number line like the circle model?



MATH PRACTICE

## Prompting Discussions with Models

What do you notice? What do you wonder?

Photos are available through the Number Talk Images website:

<http://ntimages.weebly.com/>

## What can we do?

- Step back and let them think.
- Pose problems.
- Explore through models.
- Ask deep questions.
- Encourage talk.

How we teach is as important as what we teach!

For more ideas:



Let Your Students Do the Thinking!

<http://www.Heinemann.com/mathinpractice>



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