

# Making Sense of Mathematics Through Models, Discussion, and Discovery

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## What does it mean to be mathematically proficient?

A focus on:

- Conceptual understanding
- Computational fluency
- Application (problem solving)

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

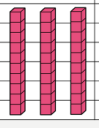
These standards view mathematicians as thinkers, communicators, reasoners, and problem solvers.

## Recognizing Patterns in Fact Sets

### Read Toasty Toes by Michael Dahl

**USE BASE-TEN BLOCKS TO MODEL AND CREATE EQUATIONS FOR 1-8 CHILDREN.**

# children	# toes on each	Multiplication equation
1	10	$1 \times 10 = 10$
2	10	$2 \times 10 = 20$
3	10	$3 \times 10 = 30$



**RECORD AND OBSERVE DATA**


# children	# toes on each	Multiplication equation
1	10	$1 \times 10 = 10$
2	10	$2 \times 10 = 20$
3	10	$3 \times 10 = 30$
4	10	$4 \times 10 = 40$
5	10	$5 \times 10 = 50$
6	10	$6 \times 10 = 60$
7	10	$7 \times 10 = 70$
8	10	$8 \times 10 = 80$
9	10	$9 \times 10 = 90$
10	10	$10 \times 10 = 100$

What is the rule?  
How can this help you multiply by ten?

## Round a 3-digit Number to the Nearest Hundred

178 people attended the chorus concert. About how many people came to the concert?

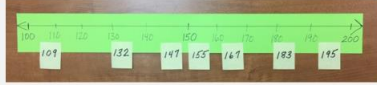
**ROUNDING TO THE NEAREST HUNDRED**



Label the other marks on your number line.  
Discuss each of the numbers on your sticky notes.  
Place each one about where it belongs on the number line.


MATH-PRACTICE

**BUILDING ON PLACE VALUE**



MATH-PRACTICE

**ROUNDING NUMBERS**



What do you notice?  
Where would you place 160? Why?  
Check it on the number line. Were you right?  
Could you decide which was the closer hundred if you didn't have a number line? How?  
What is the rule for rounding to the nearest hundred?

MATH-PRACTICE

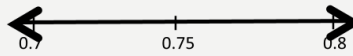
## Extending the Concept of Rounding Numbers

### WHAT'S THE RULE?

- Draw a number line showing 2,000 – 3,000, showing each hundred along the line.
- What is the midpoint? Why is it 2,500?
- Write 10 numbers that fall between 2000 and 3000.
- Where do they belong on your line?
- Round each number to the nearest thousand and place it on a chart.

Nearer to 2000	Nearer to 3000

### COULD YOU USE THE SAME TECHNIQUE TO ROUND 0.78 TO THE NEAREST TENTH?



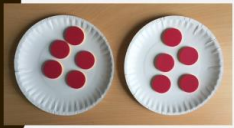
Nearer to 0.7	Nearer to 0.8

## Understanding the Commutative Property

### INVESTIGATING THE COMMUTATIVE PROPERTY WITH MODELS

2 plates of 5 cookies

5 plates of 2 cookies



$$2 \times 5 = 10$$



$$5 \times 2 = 10$$

### INVESTIGATING THE COMMUTATIVE PROPERTY WITH MODELS

- 2 plates of 5 brownies  $2 \times 5 = 10$
- 5 plates of 2 brownies  $5 \times 2 = 10$
- 2 plates of 3 brownies  $2 \times 3 = 6$
- 3 plates of 2 brownies  $3 \times 2 = 6$
- 2 plates of 4 brownies  $2 \times 4 = 8$
- 4 plates of 2 brownies  $4 \times 2 = 8$

What do you notice?

Do you think it will always happen? Why or why not?

If  $2 \times 6 = 12$ , what is the product of  $6 \times 2$ ? Try it.

What is the rule?

## Adding Fractions with Like Denominators

JANE CUT A LOAF OF BREAD INTO SIX EQUAL PIECES. SHE ATE ONE SIXTH AND HER BROTHER ATE 2 SIXTHS. HOW MANY SIXTHS DID THEY EAT?



1 sixth and 2 sixths = 3 sixths

$$\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$$

Pose similar problems and record data.

$$\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$$

$$\frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

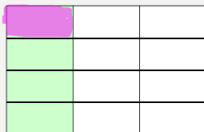
$$\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

- What do you notice?
- Does it make sense?
- Can you predict the sum of  $\frac{4}{6} + \frac{1}{6}$ ?
- Try it. Were you right?
- How did you know? Will it always work? Why or why not?
- What is the sum of  $\frac{4}{6} + \frac{2}{6}$ ?
- Does that make sense? Why?
- What is the rule for adding fractions?

## Dividing a Unit Fraction by a Whole Number

### EXPLORING FRACTION DIVISION

Four friends have  $\frac{1}{3}$  pan of brownies to share. How much of the pan will each friend get if they share it equally?



$$\frac{1}{3} \div 4 = n$$

Create a model to show the problem.

$$\frac{1}{3} \div 4 = \frac{1}{12}$$

### TRY A FEW MORE, RECORD, OBSERVE

$$\frac{1}{3} \div 4 = \frac{1}{12}$$

$$\frac{1}{3} \div 3 = \frac{1}{9}$$

$$\frac{1}{4} \div 2 = \frac{1}{8}$$

What do you notice?

Predict  $\frac{1}{4} \div 4$ .

Create a model to check your prediction.

Why is this happening?

What is the rule?

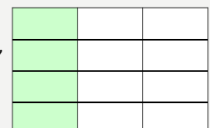
### EXPLORING FRACTION DIVISION

Have we seen something like this before?

When we divide  $\frac{1}{3}$  by 4, we are doing the same thing as when we find  $\frac{1}{4}$  of it!

How does the model help this make sense?

So,  $\frac{1}{4}$  of a quantity is the same as dividing it by 4!



$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

$$\frac{1}{3} \div 4 = \frac{1}{12}$$

### **What do these tasks have in common?**

- Visual
- Context
- Deep questioning
- Math talk
- Ask students to make sense of ideas
- Ask students to generalize
- Focus on discovery vs telling

### **Discovery vs Telling**

What are the advantages of letting students discover math rules?

### **Questions**

- What do you notice?
- Why is it happening?
- Does it make sense?
- Can you predict...?
- What is the rule?

### **Observing Models and Data**

- Do students create the models and generate the data? Why is this important?
- How is the data organized and displayed? Why does that matter?
- Can students predict and generate next data? What does that indicate?
- Can students verbalize a generalization or rule based on their observations?

**How will our students learn to be mathematical thinkers if we don't give them ongoing opportunities to do the thinking?**

**Let's step back and let them think!**

## Teacher Resource Books by Sue O'Connell

Published by Heinemann ([www.heinemann.com](http://www.heinemann.com))

### ***Math in Practice*** ([www.mathinpractice.com](http://www.mathinpractice.com))

This series is filled with lesson ideas, instructional strategies, practice tasks, and many online printable resources to make teaching K-5 math more meaningful and more fun. There is a book for each grade level K-5 that contains a wealth of grade-specific activities, as well as a *Guide for Teachers* filled with instructional strategies and an *Administrator's Guide*. Visit the website or [www.mathinpractice](http://www.mathinpractice) to view the materials.

### ***Putting the Practices into Action - Implementing the Common Core Standards for Mathematical Practice K-8*** with John SanGiovanni

The Standards for Math Practice are the heart and soul of the Common Core State Standards. This book explains each standard in teacher-friendly terms and highlights practical activities to make the standards come alive in classrooms. It contains PLC study group questions and online resources.

### ***Mastering the Basic Math Facts for Addition and Subtraction***

### ***Mastering the Basic Math Facts for Multiplication and Division***

with John SanGiovanni

Through investigations, discussions, visual models, children's literature, and hands-on explorations, students explore the math operations, and through engaging, interactive practice achieve fluency with basic facts. A teacher-friendly CD filled with customizable activities, templates, recording sheets, and teacher tools simplifies your planning and preparation. Over 450 pages of reproducible forms are included in English and Spanish translation.

### ***The Math Process Standards Series***

Each book in this series is a practical guide for helping students refine their skills in the highlighted math process (problem solving, communication, reasoning, representations, connections). You will find specific teaching strategies and tips to help all students strengthen their skills. Included with each book is a CD filled with teacher tools and customizable student activities to allow you to change names, data, or spacing for a quick way to differentiate instruction within your classroom.

***Introduction to Problem Solving***

***Introduction to Communication***

***Introduction to Representation***

***Introduction to Reasoning and Proof***

***Introduction to Connections***

All books in this series are available for Grades PK-2, Grades 3-5, and Grades 6-8.

### ***Now I Get It: Strategies for Building Confident and Competent Mathematicians, K-6***

Good teaching is the critical factor that helps students "get" math. This book is a practical handbook for the teaching of mathematics, with chapters addressing the teaching of problem solving, the use of manipulatives, differentiating instruction, effective teacher questioning, increasing math talk, and much more. The book includes a CD with over 100 pages of resources to support teachers including manipulative templates, math facts game templates, a bibliography of math-related literature, center ideas, math websites, problem-solving and writing tasks, and a variety of other practical resources.

For additional resources, visit Sue's website at [www.qualityteacherdevelopment.com](http://www.qualityteacherdevelopment.com)

Follow Sue on Twitter @SueOConnellMath

Like our Facebook page – Quality Teacher Development

Join the Math in Practice Facebook group!

More math resources: [www.heinemann.com/authors/1415.aspx](http://www.heinemann.com/authors/1415.aspx)