

Math in Practice:
Making Sense of Fractions Through Models, Discussion, and Discovery (Grades 3-5)
Woodward Academy Summit for Transformative Learning 2018

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What does proficiency with fractions look like?

Blake is making brownies and needs $\frac{3}{4}$ cup of milk, but there is no measuring cup that says $\frac{3}{4}$. How can she be sure she has just the right amount of milk?



Molly, Liam, and Bailey ordered a pepperoni pizza and they ate the whole thing!
Liam ate more than the others.
What fraction of the pizza could each child have eaten? Justify your answers.

Katie sold 30 boxes of Girl Scout Cookies.

$\frac{2}{5}$ of them were Thin Mints.

How many boxes of Thin Mint Cookies did she sell? Explain how you know.

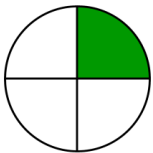
Our goal is for our students to develop a comprehensive, multi-faceted knowledge of fractions, not simply to memorize a series of procedures.

What can we do?

- Step back and let them think.
- Pose problems.
- Explore through models.
- Make connections between models.
- Ask deep questions.
- Encourage math talk.

Introducing Fraction Notation

What part of this circle is shaded green?
"a fourth"

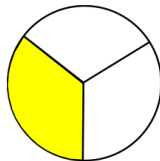


$$\frac{1}{4}$$

Turn and talk:
What do you think each number represents? Explain your thinking.

Ask, Don't Tell

What fraction would you write to show the part of this circle that is shaded yellow?
Explain the numbers in your fraction.



Let Them Try It



Write the fraction that shows the part of this circle that is colored green.
What does each number in your fraction represent?

Moving Beyond Circles and Rectangles

Can you talk about fractions using pattern blocks?

Discovering Equivalence

Paper Folding/Equivalent Fractions

Begin with a piece of $8\frac{1}{2} \times 11$ paper.

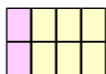
- Fold your paper in half. How would you describe one half of your paper? ($\frac{1}{2}$)
- Fold it in half again. How would you describe that same half of your paper? ($\frac{2}{4}$)
- Fold it in half again. How would you describe that same half of your paper? ($\frac{4}{8}$)
- Fold it in half again. How would you describe that same half of your paper? ($\frac{8}{16}$)

Has the size of $\frac{1}{2}$ of your paper changed?

What do you notice about each fraction? How are the numerators and denominators connected?

What other fractions might show $\frac{1}{2}$?

Equivalent Fractions



- Draw a rectangle.
- Split it into fourths with vertical lines.
- Shade $\frac{1}{4}$.
- Divide the rectangle again with a horizontal line.
- *What fractional part is shaded?*
- *Write an equality to show it.*

$$\frac{1}{4} = \frac{2}{8}$$

Record on the board.

$$\frac{1}{4} = \frac{2}{8}$$

$$\frac{1}{4} = \frac{3}{12}$$

$$\frac{1}{4} = \frac{4}{16}$$

$$\frac{1}{4} = \frac{5}{20}$$

What do you notice?

What do you wonder?

Will it always happen?

A Further Investigation

Give each team a unit fraction to model and then find equivalent fractions (e.g., $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$)

Have teams present their findings.

What do you notice?

Why is that happening?

Predict...

What is the rule? Is there a way you could find the equivalent fractions without creating the models? Explain.

Will it work for non-unit fractions?

Give each team a non-unit fraction to model and then find equivalent fractions (e.g., $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{3}{8}$)

Have teams present their findings.

What do you notice?

Does it make sense?

How can you tell if fractions are equivalent? What is always true for equivalent fractions?

What is the rule for generating equivalent fractions?

Talk About It/Write About It

What does it mean for fractions to be equivalent?

Explain to a friend how to generate an equivalent fraction. Share an example to help your friend understand.

Agree or disagree?

$$\frac{1}{5} = \frac{5}{20}$$

Justify your answer.

Equivalency Puzzles

Make a 3x3 array using 9 square sticky notes.

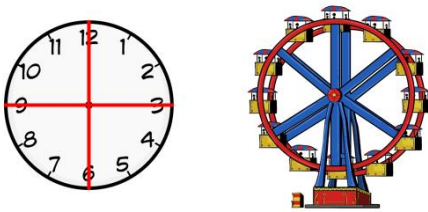
Create a puzzle by using fractions or equations with fractions for which adjacent pieces are equal.

Place values on all sides of each card to serve as distractors.

Shuffle your cards and ask a neighboring table to reassemble your puzzle.

How could you modify this to make it easier or harder for your students?

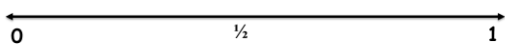
Connecting Area Models to Length/Measurement Models



Creating Number Lines from Circles

- Mark a paper circle to show fourths or eighths.
- What does each section in your circle represent?
- How are the sections for fourths different than those for eighths?
- Wrap a string (or wikki stick) around the plate and make marks to show each section.
- Unwrap the string and tape it in a line on your desk.
- Use sticky notes to label the sections of the number line.
- What do you notice?
- How are the sections for fourths different than those for eighths?

Thinking About Context Number Line Problems



Make a 0-1 number line on a sentence strip to show 0-1 miles. Place sticky notes to show the answers to the following:

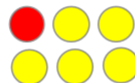
- Devon ran $\frac{1}{2}$ mile. Show how far she ran.
- Diane ran $\frac{3}{4}$ mile. Show how far she ran.

Justify your placements.

Thinking About Context

Liam had 6 jelly beans.

One was cherry and 5 were lemon.



What fraction of his jelly beans were cherry?

What fraction of his jelly beans were lemon?

Fraction of the Day

What does $\frac{3}{4}$ look like?

Students create a model to show their understanding, then compare and discuss their models.

Visualizing Adding Fractions with Unlike Denominators



Adding
Fractions with
Unlike
Denominators



Retell the problem.

What are we trying to find out?

What operation would you use? Why?

What equation represents the story?

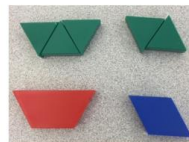
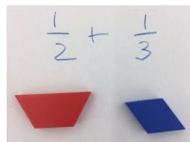
$$\frac{1}{2} + \frac{1}{3} = n$$

What prior skills would students need?

How might visualizing it help you solve it?

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Visualizing Common Denominators



Estimate a Sum or Difference

More or less than 1? Explain.

$$\begin{array}{r} \frac{4}{5} + \frac{1}{2} \\ \frac{1}{4} + \frac{1}{3} \\ 1\frac{1}{2} - \frac{3}{4} \\ 2 - \frac{5}{6} \end{array}$$

Multiplying Fractions

How would you visualize this problem?

There was $\frac{1}{2}$ cake on the kitchen counter. Kellen ate $\frac{1}{4}$ of it. How much of the whole cake did Kellen eat?



$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

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Literature Links

Full House by Dayle Ann Dodds

Picture Pie by Ed Emberley

Prompting Discussions with Models

What do you notice? What do you wonder?

Photos are available through the Number Talk Images website:

<http://ntimages.weebly.com/>

How we teach is as important as what we teach!

Teacher Resource Books by Sue O'Connell
Published by Heinemann (www.heinemann.com)

***Math in Practice* (www.mathinpractice.com)**

This series is filled with lesson ideas, instructional strategies, practice tasks, and many online printable resources to make teaching K-5 math more meaningful and more fun. There is a book for each grade level K-5 that contains a wealth of grade-specific activities, as well as a *Guide for Teachers* filled with instructional strategies and an *Administrator's Guide*. Visit the website or www.mathinpractice.com to view the materials.

***Putting the Practices into Action - Implementing the Common Core Standards for Mathematical Practice K-8*
with John SanGiovanni**

The Standards for Math Practice are the heart and soul of the Common Core State Standards. This book explains each standard in teacher-friendly terms and highlights practical activities to make the standards come alive in classrooms. It contains PLC study group questions and online resources.

***Mastering the Basic Math Facts for Addition and Subtraction*
Mastering the Basic Math Facts for Multiplication and Division
with John SanGiovanni**

Through investigations, discussions, visual models, children's literature, and hands-on explorations, students explore the math operations, and through engaging, interactive practice achieve fluency with basic facts. A teacher-friendly CD filled with customizable activities, templates, recording sheets, and teacher tools simplifies your planning and preparation. Over 450 pages of reproducible forms are included in English and Spanish translation.

The Math Process Standards Series

Each book in this series is a practical guide for helping students refine their skills in the highlighted math process (problem solving, communication, reasoning, representations, connections). You will find specific teaching strategies and tips to help all students strengthen their skills. Included with each book is a CD filled with teacher tools and customizable student activities to allow you to change names, data, or spacing for a quick way to differentiate instruction within your classroom.

Introduction to Problem Solving

Introduction to Communication

Introduction to Representation

Introduction to Reasoning and Proof

Introduction to Connections

All books in this series are available for Grades PK-2, Grades 3-5, and Grades 6-8.

Now I Get It: Strategies for Building Confident and Competent Mathematicians, K-6

Good teaching is the critical factor that helps students "get" math. This book is a practical handbook for the teaching of mathematics, with chapters addressing the teaching of problem solving, the use of manipulatives, differentiating instruction, effective teacher questioning, increasing math talk, and much more. The book includes a CD with over 100 pages of resources to support teachers including manipulative templates, math facts game templates, a bibliography of math-related literature, center ideas, math websites, problem-solving and writing tasks, and a variety of other practical resources.

For additional resources, visit Sue's website at www.qualityteacherdevelopment.com

Follow Sue on Twitter @SueOConnellMath

Like our Facebook page – Quality Teacher Development

Join our *Math in Practice* Facebook group!